

Differential Equation (Orthogonal trajectory)

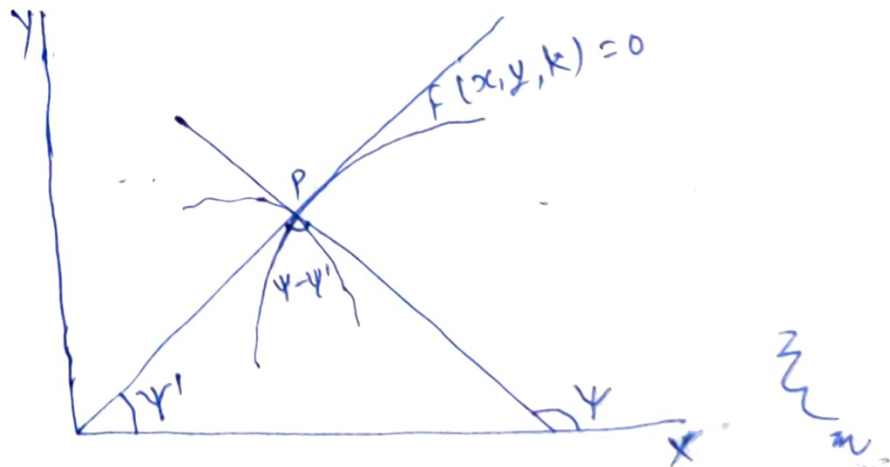
A curve which cuts every member of a given family of curves according to a given rule is called a trajectory of the given family. We shall be concerned only with the case when each trajectory cuts every member of a given family of curves at a constant angle.

If the constant angle is a right angle, then the trajectory is said to be orthogonal.

Rule for finding the orthogonal trajectory to a given family of curves when its equation is given in -

1. Cartesian Co-ordinates

To find the orthogonal trajectories of the family of curves $f(x, y, c)$; c being a parameter.



The equation of the curve is given to be

$$f(x, y, c) = 0 \quad \text{--- (1)}$$

Differentiating (1) w.r.t. x and eliminating c , between (1) and the derived result, we shall obtain the differential equation of the given

family of curves. The differential equation obtained will evidently involve x, y and $\frac{dy}{dx}$.

Let the differential equation be

$$\phi\left(x, y, \frac{dy}{dx}\right) = 0 \quad \text{--- --- --- } (2)$$

Let $F(x, y, k) = 0$ be the eqn. of the curve which cuts the given family $f(x, y, c) = 0$ at a constant angle. We know that the angle between the curves is equal to the angle between the tangents at their point of intersection.

Let P be the point of their intersection.

Let $m (= \tan \psi = \frac{dy}{dx})$ and m' denote the slope of the tangents to the curves $f(x, y, c) = 0$ and $F(x, y, k) = 0$ respectively.

$$\text{Then } \tan \alpha = \frac{m - m'}{1 + mm'}$$

If $\alpha = 90^\circ$, i.e. if the trajectories are orthogonal, then $1 + mm' = 0$.

$$\therefore mm' = -1 \Rightarrow m' = -\frac{1}{m} = -\frac{1}{\frac{dy}{dx}} = -\frac{dx}{dy}$$

Thus if $\frac{dy}{dx}$ is the slope of the tangent of the given family of curves, then the slope of the trajectories would be $-\frac{dx}{dy}$.

Hence the differential equation of the family (of curves), the of orthogonal trajectories; from (2), it

$$\phi\left(x, y, -\frac{dx}{dy}\right) = 0.$$